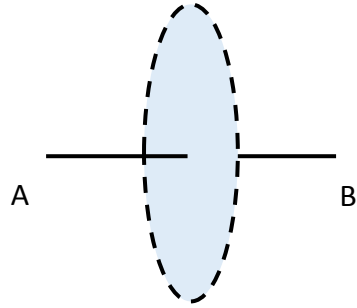


# 107 學年度學科能力測驗

## 數學考科解析

1. 唯有此圓上之點 P，方能滿足  $\overline{PA} = \overline{PB} = \overline{AB}$



2. 每題選對機率  $1/5$

$$10 \text{ 題} \Rightarrow \left(\frac{1}{5}\right)^{10} = \frac{1}{5^{10}}$$

$$\log 5^{10} = 10 \log 5 \approx 10 \times 0.699 = 6.99 \approx 7$$

$$\text{故 } \frac{1}{5^{10}} \text{ 最接近 } \frac{1}{10^7} = 10^{-7}$$

3. 符合題目的情況=全部情況扣掉-(甲先任挑兩天，乙再從剩下五天裡挑兩天)

$$\text{故 } P = 1 - \frac{C_2^7 C_2^5}{C_2^7 C_2^7} = 1 - \frac{10}{21} = \frac{11}{21}$$

4. 三數取對數：

$$\log 10^9 < \log 2^x < \log 9^{10}$$

$$\Rightarrow 9 < x \log 2 < 10 \log 9 = 9.542$$

$$\therefore \frac{9}{\log 2} < x < \frac{9.542}{\log 2} \Rightarrow 29.9 < x < 31.7$$

故  $x = 30, 31$ ，故兩個

5. 等差數列  $\Rightarrow \cos(3\theta - 60^\circ) + \cos(3\theta + 60^\circ) = 2 \cos \theta$

$$\Rightarrow \cos 3\theta \cos 60^\circ + \sin 3\theta \sin 60^\circ + \cos 3\theta \cos 60^\circ - \sin 3\theta \sin 60^\circ = 2 \cos 3\theta$$

$$\Rightarrow 2 \cos 3\theta \times \frac{1}{2} = 2 \cos 3\theta$$

$$\Rightarrow \cos 3\theta = 0$$

$$\text{又 } 0^\circ < \theta < 180^\circ \Rightarrow 0^\circ < 3\theta < 540^\circ \Rightarrow 3\theta = 90^\circ \text{ or } 270^\circ \text{ or } 450^\circ$$

故  $\theta = 30^\circ \text{ or } 90^\circ \text{ or } 150^\circ$

爾雅數位

$$6. \quad x = \text{上週售價} \times \{ \text{售價漲跌幅} + 1 \} = 180 \left\{ \frac{1}{2} \left( \frac{50-100}{100} \right) + 1 \right\} = 135$$

$$y = x \left\{ \frac{1}{2} \left( \frac{90-50}{50} \right) + 1 \right\} = x \times 1.4 = 189$$

故答案選(5)

$$7. \quad \overline{OA} + \overline{OB} + \sqrt{3}\overline{OC} = \vec{0}$$

$$\Rightarrow \overline{OB} + \sqrt{3}\overline{OC} = -\overline{OA}$$

$$\Rightarrow |\overline{OB} + \sqrt{3}\overline{OC}|^2 = |-\overline{OA}|^2$$

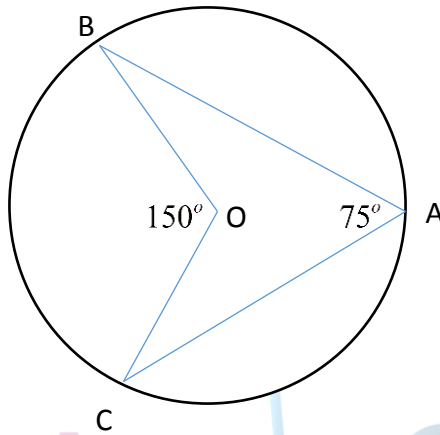
$$\Rightarrow |\overline{OB}|^2 + 2\sqrt{3}\overline{OB} \cdot \overline{OC} + 3|\overline{OC}|^2 = |\overline{OA}|^2$$

$$\Rightarrow 1^2 + 2\sqrt{3} \times 1 \times 1 \times \cos \angle BOC + 3 \times 1^2 = 1^2$$

$$\Rightarrow \cos \angle BOC = \frac{-\sqrt{3}}{2}$$

$$\text{故 } \angle BOC = 150^\circ$$

$$\Rightarrow \angle BAC = \frac{1}{2} \angle BOC = 75^\circ$$



8. 小華的成績：國文均標、英文前標、數學均標、自然均標、社會前標

(1) 皆達成標準

(2) 國文未達成前標

(3) 數學、自然皆未達成前標

(4) 英文達成前標，且自然、社會均達成均標，故達成標準

(5) 自然未達前標

故選(1)、(4)

$$9. \quad f(x) = (x^2 - 1) \cdot q(x) + 2x + 1$$

$$(1) f(0) = -q(0) + 1$$

$$(2) f(1) = 2 + 1 = 3$$

(3) if  $q(x) = 0$ , 則  $f(x)$  為一次式

$$(4) 4x^4 + 2x^2 - 3 = (x^2 - 1)(4x^2 + 6) + 3, \text{ 不合}$$

$$(5) 4x^4 + 2x^3 - 1 = (x^2 - 1)(4x^2 + 2x + 4) + 2x + 1$$

故選(2)、(3)、(5)

10. (1)  $\overline{BC} = \overline{AC} - \overline{AB} = \left(\frac{22}{5}, -\frac{11}{5}\right)$ , 故  $\overline{BC} = \frac{11}{5}\sqrt{5}$

(2)  $\overline{AC} \cdot \overline{BC} = 0$

(3)  $\because C$  為直角, 故  $\Delta ABC = \frac{\overline{AC} \times \overline{BC}}{2} = \frac{2\sqrt{5} \times 11\sqrt{5}}{2 \times 5} = \frac{11}{5}$

(4)  $\sin B = \frac{2\sqrt{5}}{25} < \sin C = 1$

(5)  $\cos A = \frac{2\sqrt{5}}{25} < \cos B = \frac{11\sqrt{5}}{25}$

11. (1) 錯誤;  $(3, 0, -1)$  代入  $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z}{-1} \Rightarrow 1 = \frac{2}{3} = 1$  不合

(2) 錯誤;  $(1, 2, 3)$  代入  $E_1: 2x-3y-z=0$ , 不合

(3) 正確;  $L$  的方向向量  $(2, -3, -1) // E_1$  的法向量  $(2, -3, -1)$ , 故  $L \perp E_1$

(4) 錯誤;  $L$  上取一點  $(1, 2, 0)$  代入  $E_2: x+y-z=0$ , 不合

(5) 正確;  $E_1$  的法向量  $(2, -3, -1)$  不平行  $E_2$  法向量  $(1, 1, -1)$ , 故  $E_1$  與  $E_2$  交於一線  
故選(3)、(5)

12.

	方程式	a	b	c	焦點
題幹	$y^2 = 2x$	/			$\left(\frac{1}{2}, 0\right)$
(1)	$y = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4}$				$\left(\frac{1}{2}, 0\right)$
(2)	$\frac{x^2}{4} + \frac{y^2}{3} = 1$	2	$\sqrt{3}$	1	$(\pm 1, 0)$
(3)	$x^2 + \frac{4y^2}{3} = 1$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\left(\pm \frac{1}{2}, 0\right)$
(4)	$8x^2 - 8y^2 = 1$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{2}$	$\left(\pm \frac{1}{2}, 0\right)$
(5)	$4x^2 - 4y^2 = 1$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\left(\pm \frac{\sqrt{2}}{2}, 0\right)$

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A. 同一直線，斜率相同

$$\therefore \frac{\log 6 - \log 3}{6 - 3} = \frac{y - \log 6}{12 - 6}$$

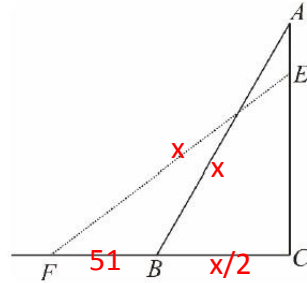
故  $y = \log 24$

B. 令  $\overline{AB} = x$ ，則  $\overline{BC} = x \cos 60^\circ = \frac{1}{2}x$

又  $\sin \angle EFC = 0.6 \Rightarrow \cos \angle EFC = 0.8$

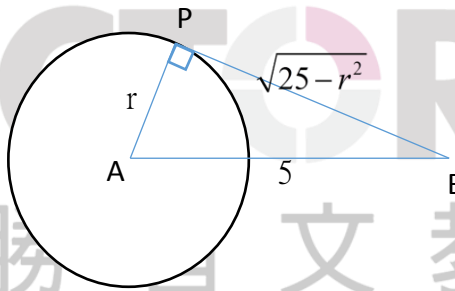
$\triangle CEF$  中， $\overline{EF} = x$ ， $\overline{FC} = \frac{1}{2}x + 51$

$$\cos \angle EFC = \frac{\overline{FC}}{\overline{EF}} \Rightarrow 0.8 = \frac{\frac{1}{2}x + 51}{x} \Rightarrow x = 170$$



C.  $\Delta PAB = \frac{1}{2}r\sqrt{25-r^2} = \frac{1}{2}\sqrt{r^2(25-r^2)} \leq \frac{1}{2} \times \frac{r^2 + (25-r^2)}{2} = \frac{25}{4}$

(算幾不等式)

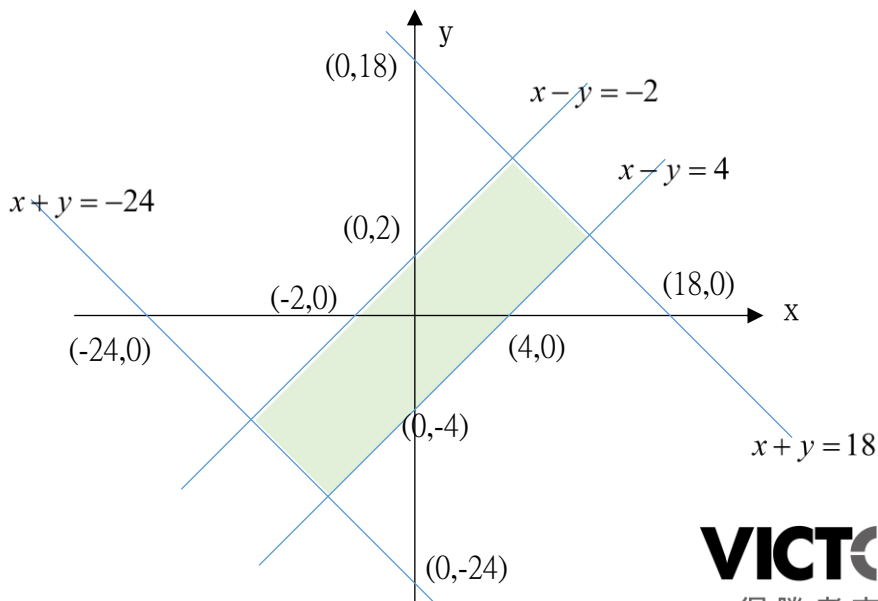


爾雅數位

D. 四條線圍成的四方形內部

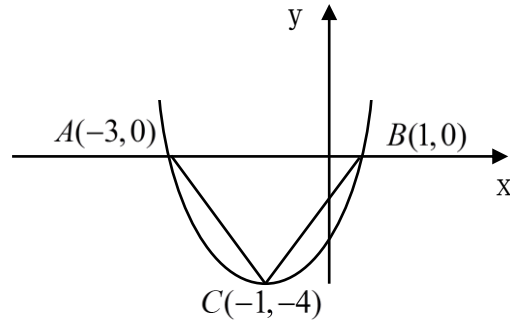
最大直徑即為  $x-y=4$  至  $x-y=-2$  兩直線的距離  $= \frac{|-2-4|}{\sqrt{2}} = \frac{6}{\sqrt{2}}$

$\rightarrow$  半徑  $= \frac{3}{\sqrt{2}}$ ，面積  $= (\frac{3}{\sqrt{2}})^2 \pi = \frac{9}{2} \pi$



E.  $\overline{AC} = \sqrt{20}, \overline{BC} = \sqrt{20}, \overline{AB} = 4$

$$\cos \angle ABC = \frac{\sqrt{20}^2 + \sqrt{20}^2 - 4^2}{2 \times \sqrt{20} \times \sqrt{20}} = \frac{3}{5}$$



F.  $[1 \ 2] \begin{bmatrix} 7 \\ e \end{bmatrix} = [23]$ , 故  $e=8$

$$[c \ d] \begin{bmatrix} -3 & 7 \\ -4 & 8 \end{bmatrix} = [0 \ 7], \text{ 故 } c=7, d=\frac{-21}{4}$$

$$[y] = [c \ d] \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

G. 如圖，

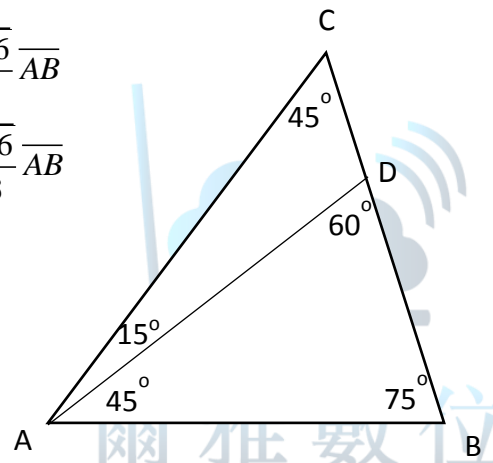
$$\Delta ABC \text{ 中，由正弦定理知 } \frac{\overline{BC}}{\sin 60^\circ} = \frac{\overline{AB}}{\sin 45^\circ} \Rightarrow \overline{BC} = \frac{\sqrt{6}}{2} \overline{AB}$$

$$\Delta ABD \text{ 中，由正弦定理知 } \frac{\overline{BD}}{\sin 45^\circ} = \frac{\overline{AB}}{\sin 60^\circ} \Rightarrow \overline{BD} = \frac{\sqrt{6}}{3} \overline{AB}$$

$$\overline{CD} = \overline{BC} - \overline{BD} = \frac{\sqrt{6}}{2} \overline{AB} - \frac{\sqrt{6}}{3} \overline{AB} = \frac{\sqrt{6}}{6} \overline{AB}$$

$$\Rightarrow \overline{CD} : \overline{DB} = \frac{\sqrt{6}}{6} \overline{AB} : \frac{\sqrt{6}}{3} \overline{AB} = 1 : 2$$

$$\text{由分點公式得知 } \overline{AD} = \frac{1}{3} \overline{AB} + \frac{2}{3} \overline{AC}$$



H.  $\overline{DF} = \overline{BE} = 15 \times \frac{3}{5} = 9$

$$\overline{EF} = 25 - 9 \times 2 = 7$$

$$\overline{CE} = \sqrt{12^2 + 7^2} = \sqrt{193}$$

$$\overline{AC} = \sqrt{144 + 193} = \sqrt{337}$$

